OCR
Oxford Cambridge and RSA

GCE

Mathematics A

H240/01: Pure Mathematics

A Level

Mark Scheme for June 2023

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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MARKING INSTRUCTIONS

PREPARATION FOR MARKING
RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training*; *OCR Essential Guide to Marking*.

2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal http://www.rm.com/support/ca

3. Log-in to RM Assessor and mark the **required number** of practice responses ("scripts") and the **number of required** standardisation responses.

MARKING

1. Mark strictly to the mark scheme.

2. Marks awarded must relate directly to the marking criteria.

3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. **Annotations**

Annotation	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

5. **Subject Specific Marking Instructions**

a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)
- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
- When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is not given in the paper accept any answer that agrees with the correct value to 3 s.f. unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g. Rules for replaced work and multiple attempts:
- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1	(a)		$BC^2 = 6^2 + 15^2 - 2\times6\times15\times\cos30^o$	M1	1.1a	Attempt use of cosine rule	Allow either omission of 2, or + not –, but no other errors Allow other fully complete methods, such as basic trigonometry, possibly combined with Pythagoras
			$BC = 10.3$ cm	A1	1.1	Obtain 10.3cm, or better	If > 3sf then allow 10.25, or answers that round to 10.25 Condone no units
				[2]			
1	(b)		$\frac{\sin 30}{4} = \frac{\sin D}{6}$	M1	1.1	Attempt use of sine rule	Correct equation seen, with fractions either way up Could also be implied by method eg $\sin^{-1}(0.75)$ is **M1**, but just 0.75 is **M0** Allow other fully complete methods
			$D = 48.6^o$	A1	1.1	Obtain $D = 48.6^o$, or better	D = 48.590377… Allow D = 0.848 radians
			or $D = 131^o$	A1FT	3.1a	Obtain $D = 131^o$, or better FT their first angle as long as $<150^o$	D = 131.409622… **A0** if additional angles given as well Allow D = 2.29 radians (could be FT on incorrect acute angle in radians, as long as $D < 2.618$)
				[3]			

Question			Answer	Marks	AO	Guidance	
2	(a)	(i)	$\frac{3+2\sqrt{x}+3-2\sqrt{x}}{\left(3-2\sqrt{x}\right)\left(3+2\sqrt{x}\right)}$	M1	1.1	Attempt to rewrite fractions using correct common denominator	Common denominator could just appear as $9 - 4x$ Must include correct attempt at numerators as well
			$\frac{6}{9-4x}$	A1 [2]	2.1	Obtain correct simplified fraction	No need to state values for a, b and c explicitly www – if middle terms shown for expansion of denominator, then these must be correct ISW any further attempt to 'simplify' **SC B1** for answer only, with no method shown
2	(a)	(ii)	$\frac{6}{9-4x}=2$ $6 = 18 - 8x$ $8x = 12$	M1	1.1a	Attempt to solve equation – as far as clearing the fraction and combining constant terms	**M1** for using their fraction, as long as of correct form Correct method to clear fraction, so **M0** for eg $6 = 18 - 4x$, but allow sign error when combining constant terms
			$x = \frac{3}{2}$	A1 [2]	1.1	Obtain $x = \frac{3}{2}$	aef, but fractions must be simplified

Question			Answer	Marks	AO	Guidance	
2	(b)		**DR** $\left(2^y-8\right)\left(2^y+1\right)$	**M1**	**3.1a**	Attempt to solve disguised quadratic in 2^y	If factorising then expansion should give x^2 and one other term correct Quadratic formula should be correct – allow one slip when substituting as long as general formula already seen as correct Completing the square needs to go as far as $x-p=\pm\sqrt{q}$
			$2^y=8$, $2^y=-1$	**A1**	**2.1**	Obtain two correct roots (could still be in terms of eg u if substitution used)	**SC** If no method shown then award **B1** in place of **M1A1** for both correct roots (final two marks can still be awarded)
			$y=\log_2 8=3$	**M1**	**1.1**	Attempt to solve $2^y=k$, where $k>0$ May just see $y=3$, with no explicit use of $\log_2$	Allow BOD if attempt at solving $2^y=-1$ still present If $k\neq 8$ then solution method must be seen, even if k is a power of 2
			$y=3$ only; $2^y=-1$ has no solutions as $2^y>0$ for all y	**A1**	**2.3**	Obtain $y=3$, having rejected $2^y=-1$ with some reasoning	Must have some reason, eg '2^y is always positive', '2^y cannot be negative', 'cannot take log of a negative number', 'not defined', 'not real', 'no solutions' **A0** for 'math error', 'does not work', 'not possible'
				[4]			**SC** If no method at all shown then allow **B1** for $y=3$, with no other solutions

Question			Answer	Marks	AO	Guidance	
3	(a)		$f(x+h) - f(x) =$ $((x+h)^2 + 2(x+h)) - (x^2 + 2x)$	**M1**	**2.1**	Attempt expression for $f(x+h) - f(x)$	Allow sign error from no bracket around final term, ie $(x+h)^2 + 2(x+h) - x^2 + 2x$ is **M1**, but no other errors allowed If considering x^2 and $2x$ separately then expressions for both must be seen
			$= x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x$ $= 2xh + h^2 + 2h$	**M1**	**2.1**	Expand and simplify $f(x+h) - f(x)$	Expand and gather like terms (either separately, or single expression) Condone sign errors only, so M0 if collecting like terms after an incorrect attempt to divide by h Allow BOD if $2x\ldots + 2x$ becomes 0 rather than $4x$
			$\frac{f(x+h)-f(x)}{h} = \frac{2xh+h^2+2h}{h}$ $= 2x+h+2$	**M1**	**2.1**	Attempt $\frac{f(x+h)-f(x)}{h}$	Divide **all** terms by h Allow BOD if previous error results in a term with a denominator of h
			$f'(x) = \lim_{h\to 0}(2x+h+2) = 2x+2$	**A1**	**2.5**	Complete proof by considering limit as $h \to 0$	www, including correct signs throughout Must divide by h before $h \to 0$ Must see 'lim', '$h \to 0$', and $f'(x)$ at some point in their solution and not just when quoting the generic formula, but allow BOD for $f'(x) = \frac{f(x+h)-f(x)}{h}$ followed by =…, =…, =… on subsequent lines **A0** if 'lim' still in final answer Condone $\frac{dy}{dx}$ in place of $f'(x)$
				[4]			

Question			Answer	Marks	AO	Guidance	
3	(b)		$y = x^2 + 2x + c$	B1	2.2a	State or imply correct equation, including + c	'$y =$' could be implied by use of 5 c may be implied by later work
			$5 = 1 - 2 + c$ $c = 6$	M1	1.1	Attempt c using (– 1, 5)	Allow **M1** if equation incorrect, as long as from attempt at integrating $2x + 2$ ie of form $y = kx^2 + 2x + c$ c may be implied by method eg $y = x^2 + 2x$, followed by $5 = 1 - 2$ and then an attempt to 'balance' the sides Must use x and y the correct way around As far as attempting a value for c
			$y = x^2 + 2x + 6$	A1	1.1	Obtain correct equation, including $y = ...$	Equation must be stated, and not just implied by $c = 6$ seen Allow f(x) = … **A0** for 'equation' = $x^2 + 2x + 6$
				[3]			Just stating $y = x^2 + 2x + 6$ or $y = (x + 1)^2 + 5$ gets full marks (may come from observing that (-1, 5) is the minimum point)

Question			Answer	Marks	AO	Guidance	
4	(a)		$AB = \sqrt{2^2+4^2} = \sqrt{20} = 2\sqrt{5}$	**B1** **[1]**	**1.1**	Correct length aef	Condone 4.47 or better Allow isw eg $\sqrt{20} = 4\sqrt{5}$ Allow BOD on signs eg $AB = -2\mathbf{i} - 4\mathbf{j}$ seen
4	(b)		$(p-3)^2 + (p-5)^2 = 20$	**M1**	**1.1a**	Attempt correct equation for length BC	Using their attempt at length of AB Condone error on RHS eg having $\sqrt{20}$ not 20
			$p^2 - 8p + 7 = 0$ $p = 7$	**A1**	**1.1**	**BC** Solve correct quadratic to obtain at least $p = 7$	If second value of p stated then it must be correct
			C is $7\mathbf{i} + 7\mathbf{j}$	**A1** **[3]**	**1.1**	Correct position vector for C; it could be given as column vector, but not coordinate	No need to discard $p = 1$ If **M0**, question is 'determine' so some evidence needed for full marks – either justifying lengths are equal, or use of components of 2 and 4 $7\mathbf{i} + 7\mathbf{j}$ with some explanation **B3** $7\mathbf{i} + 7\mathbf{j}$ with no explanation **B2** (7, 7) with some explanation **B2** (7, 7) with no explanation **B1**

Question			Answer	Marks	AO	Guidance	
4	(c)		OM is $4\mathbf{i} + 4\mathbf{j}$ OR BM is $\mathbf{i} - \mathbf{j}$	**B1**	**1.1**	Correct midpoint soi Could instead find vector BM	Allow M seen as coordinate, as it is part of their method and not a requested answer Condone $M = 4\mathbf{i} + 4\mathbf{j}$, but penalise clear error eg $AM = 4\mathbf{i} + 4\mathbf{j}$ is **B0** Could be soi on a diagram
			D is $6\mathbf{i} + 2\mathbf{j}$	**B1** **[2]**	**1.1**	Correct position vector (not coordinate) for D	Do not penalise D given as coordinate if already penalised in part **(b)** Answer only is **B0B1**
4	(d)		Kite	**B1***	**2.2a**	Mark independently of reason	
				B1dep*	**2.2a**	Evidence is required to support statements made	All relevant evidence quoted must be correct
			eg two pairs of adjacent sides of same length			$AD = CD = \sqrt{26}$ (or compare components of vectors) ; condone not stating $AB = BC$ as given in question	Sides must be defined as adjacent, so **B0** for just 'two pairs of equal sides', but allow BOD if clarified on an explicit diagram seen in part **(d)**
			eg diagonals are perpendicular			AC has gradient of 1, BD has gradient of -1	If using a geometrical argument, then identify that ABC is isosceles, M is mid-point of AC hence perpendicular bisector
			eg BD being a line of symmetry			$AM = MC$, with perpendicular argument as above	
						B0 for reasoning using angles (ie a pair of facing equal angles) unless justified.	
				[2]			

Question			Answer	Marks	AO	Guidance	
5	(a)	(i)	$a = 2$	B1	1.1	Either stated or embedded in equation	eg $\|2x - b\|$ seen ignore any other values seen **B0** for $a = -2$, unless subsequently corrected
			$b = 6$	B1	1.1	Either stated or embedded in equation	eg $\|ax - 6\|$ seen ignore any other values seen
			$c = 1$	B1 [3]	1.1	Either stated or embedded in equation	eg $\|ax - b\| + 1$ seen ignore any other values seen
5	(a)	(ii)	Because f is a many to one function eg f(0) = f(6)	B1 [1]	1.2	Any correct reason	Condone no explicit example Could also say 'because f is not one to one' **B1** BOD for 'it is not one to one' If referring to 'one to many' or 'many to one' it must be clear whether this is f or f^{-1} (just 'it' or 'the function' is not enough) Allow implication of function eg 'as it is a many to one function there is no inverse function' May also refer to the 'horizontal line test', but need to state outcome eg horizontal line would cross graph of $y = f(x)$ twice

Question			Answer	Marks	AO	Guidance	
5	(b)	(i)	$y = px - q$ $px = y + q$ $x = \frac{1}{p}(y + q)$	M1	3.1a	Complete attempt to find inverse function of f(x) = $px - q$	Correct order of operations, allow sign error only Could use coordinate geometry and reflection in $y = x$ Allow **M1** BOD if more than one function is being considered
			$g^{-1}(x) = \frac{1}{p}x + \frac{q}{p}$	A1	1.1	Obtain correct inverse, in terms of x	Could be single term ie $g^{-1}(x) = \frac{x+q}{p}$ **A1** for just $\frac{1}{p}x + \frac{q}{p}$, ie $g^{-1}(x)$ can be omitted If LHS seen, it must be $g^{-1}(x)$ or y (allow BOD for g^{-1}, or using f not g) BOD if modulus sign included **A0** if additional equations given
			$x \geq 0$	B1	1.2	Correct domain **B0** for $x > 0$	Independent of the first two marks If in words then must be correct, so **B1** for 'any non-negative x' but **B0** for 'any positive x' $g^{-1}(x) \geq 0$ is **B0** Condone incorrect set notation as long as intention is clear
				[3]			
5	(b)	(ii)	$0 < p \leq 1$	B1	3.1a	Correct set of values, any notation No need for $0 < p$ as specified in question, so **B1** for $p \leq 1$	**B0** for $p < 1$ **B0** for any additional incorrect values **B0** if just single example and not set of values Condone incorrect set notation as long as intention is clear

Question			Answer	Marks	AO	Guidance	
				[1]			

Question			Answer	Marks	AO	Guidance	
6	(a)		$\frac{dy}{dx} = (2x+3)e^{x^2+3x}$	M1	1.1a	Attempt to differentiate using the chain rule	Obtain derivative of form $f(x)e^{x^2+3x}$ Could also split into two terms and use product rule to obtain derivative of form $f(x)e^{x^2}e^{3x} + ke^{x^2}e^{3x}$ $(k \neq 0)$ **M0** if attempt to split results in sum not product
			$(2x+3)e^{x^2+3x}$	A1	1.1	Obtain correct derivative	Brackets must be seen, or implied by later work aef eg $(2xe^{x^2})(e^{3x}) + (e^{x^2})(3e^{3x})$ from splitting into two terms first Could be in terms of u, as long as u clearly defined
			$(2x+3)e^{x^2+3x} = 0$ $2x + 3 = 0$ $x = -\frac{3}{2}$	A1	1.1	Equate correct derivative to 0 soi and solve to obtain $x = -\frac{3}{2}$	ISW any y-coordinates if given **A0** if any additional solutions for x Must see differentiation, so $x = -\frac{3}{2}$ with no supporting method gets no credit (as question is 'determine')

Question			Answer	Marks	AO	Guidance	
			$e^{x^2+3x} > 0$ for all x or $e^{x^2+3x} \neq 0$ or $x^2 + 3x = \ln 0$, but this is not possible	**B1FT** **[4]**	**2.4**	Indicate no solutions from the exponential term	FT their derivative as long as of form $f(x)e^{x^2+3x}$ or $f(x)e^{u}$ Allow BOD for explanations such as $e^x > 0$ for all x Must have some reason, eg 'e^{x^2+3x} is always positive', 'e^{x^2+3x} cannot be negative', 'cannot take ln of a negative number', 'not defined', 'not real', 'no solutions' **A0** for 'math error' or 'doesn't work'
6	**(a)**		**Alternative method** $\ln y = x^2 + 3x$ $\frac{1}{y}\frac{dy}{dx} = 2x + 3$	**M1**		Take ln and attempt implicit differentiation	Must deal correctly with $\ln y$
			$\frac{dy}{dx} = y(2x+3)$	**A1**		Obtain correct derivative	May still have $\frac{1}{y}$ on LHS
			$2x + 3 = 0$ $x = -\frac{3}{2}$	**A1**		Equate correct derivative to 0 soi and solve to obtain $x = -\frac{3}{2}$	
			$e^{x^2+3x} \neq 0$	**B1**		Indicate no solutions from the exponential term	See main MS for guidance Could also explain why no solutions from $\frac{1}{y}$

Question	Answer	Marks	AO	Guidance
6 (b)	$\frac{d^2y}{dx^2} = 2e^{x^2+3x} + (2x+3)^2 e^{x^2+3x}$	M1	3.1a	Attempt to differentiate again using the product rule correctly	Obtain derivative of form $(ax^2+bx+c)e^{x^2+3x}$ aef
	$\frac{d^2y}{dx^2} = \left(2+(2x+3)^2\right)e^{x^2+3x}$	A1	1.1	Obtain correct derivative	aef eg (depending on method) $2e^{x^2+3x} + 2x(2x+3)e^{x^2+3x} + 3(2x+3)e^{x^2+3x}$ or $\left(\left(2e^{x^2} + 4x^2e^{x^2}\right)e^{3x} + \left(2xe^{x^2}\right)3e^{3x}\right) +$ $\left(6xe^{x^2}e^{3x} + 9e^{x^2}e^{3x}\right)$ Could be in terms of u, as long as u clearly defined
	convex means $\frac{d^2y}{dx^2} > 0$	B1	1.2	State, or clearly imply, correct condition at any point in proof	Must be general statement, and not just > 0 from testing the stationary point

Question			Answer	Marks	AO	Guidance	
			$(2x+3)^2 \geq 0$ hence $2 + (2x+3)^2 > 0$	**M1**	**3.1a**	Explain why correct quadratic is always positive	Could note minimum value of 2 as completed square form Could use expanded quadratic, which should be $4x^2 + 12x + 11$ If showing no real roots then must also say that it is a positive quadratic Condone > / ≥ muddles for **M1** only Could show that there are no points of inflection **and** $\frac{d^2y}{dx^2} > 0$ for at least one point
			$e^{g(x)} > 0$ for all x; quadratic > 0 for all x hence curve is always convex	**A1** **[5]**	**2.4**	Full and convincing proof to show that curve is convex for all x	www
6	**(b)**		**Alt method for first 2 marks** $\frac{dy}{dx} = y(2x+3)$ $\frac{d^2y}{dx^2} = 2y + (2x+3)\frac{dy}{dx}$	**M1**		Attempt second derivative, using implicit differentiation and the product rule	If still $\frac{1}{y}\frac{dy}{dx} = 2x+3$ then must be a correct attempt to differentiate the LHS
				A1		Obtain correct derivative	aef
			Then B1 M1 A1 as above			Will need to use $\frac{dy}{dx} = y(2x+3)$ to make further progress	

Question	Answer	Marks	AO	Guidance

Question			Answer	Marks	AO	Guidance	
7	(a)		$\cos(A-B) = \cos A\cos(-B) - \sin A\sin(-B)$	M1	2.1	Replace B with $-B$ in given identity	
			$\cos(-B) = \cos B$, $\sin(-B) = -\sin B$, $\cos(A-B) = \cos A\cos B - \sin A(-\sin B)$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$ **A.G.**	A1 [2]	2.4	State $\cos(-B) = \cos B$ and $\sin(-B) = -\sin B$, and conclude with correct identity Condone $-\sin A\sin(-B)$ becoming $\sin A\sin B$ with no intermediate step	$\cos(-B) = \cos B$, $\sin(-B) = -\sin B$ must be stated, but no justification needed

Question			Answer	Marks	AO	Guidance	
7	(b)		$\left(\frac{\sqrt{3}}{2}\cos\theta-\frac{1}{2}\sin\theta\right)\left(\frac{\sqrt{3}}{2}\cos\theta+\frac{1}{2}\sin\theta\right)$	**B1**	**2.1**	Use correct identities, with exact trig values, to obtain a correct expression	Allow BOD for ambiguous positioning of + and – signs in a product, but penalise explicit errors if a single identity is seen in isolation If expansion done before exact trig values used, then the expression must still be correct at the point that the **B1** is awarded
			$\frac{3}{4}\cos^2\theta-\frac{1}{4}\sin^2\theta$	**M1**	**2.1**	Expand brackets May be recognised as difference of two squares so no need to see $\frac{\sqrt{3}}{4}\cos\theta\sin\theta-\frac{\sqrt{3}}{4}\cos\theta\sin\theta$	To obtain answer of form $a\cos^2\theta-b\sin^2\theta\,(a>0,\,b>0)$, with possibly $c\cos\theta\sin\theta-c\cos\theta\sin\theta$ also present
			$\frac{3}{4}\cos^2\theta-\frac{1}{4}\left(1-\cos^2\theta\right)$ $\cos^2\theta-\frac{1}{4}$ **A.G.**	**A1** **[3]**	**2.1**	Use Pythagorean identity and simplify to given answer	www eg if middle terms shown for expansion, then these must be correct
7	(c)	(i)	max value is $\frac{3}{4}$	**B1**	**1.1**	Correct max value	
			when θ is 180^o	**B1** **[2]**	**1.1**	Correct angle	**B0** if any extra angles given Must be 'positive' so **B0** for 0^o Must be in degrees Marks are independent
7	(c)	(ii)	min value is $-\frac{1}{4}$	**B1**	**1.1**	Correct min value	

Question			Answer	Marks	AO	Guidance	
			when θ is 90^o	**B1** **[2]**	**1.1**	Correct angle	**B0** if any extra angles given Must be in degrees **SC** If angles in **both** parts are correct, but in radians, then penalise only once (mark as **B0** in **(i)** and **B1** in **(ii)**) Marks are independent

Question			Answer	Marks	AO	Guidance	
8	**(a)**		$\left(1+\frac{3}{4}x\right)^{\frac{3}{2}} = 1+\left(\frac{3}{2}\right)\left(\frac{3}{4}x\right)$	**B1**	**1.1**	Correct first two terms	Allow unsimplified Expect $1+\frac{9}{8}x$

Question			Answer	Marks	AO	Guidance	
			$+\frac{\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)}{2}\left(\frac{3}{4}x\right)^2$	**M1**	**1.1**	Attempt third term	Condone lack of brackets when attempting to square ie $\frac{3}{4}x^2$ Coefficient must be $\frac{\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)}{2}$ or equiv
				A1	**1.1**	Obtain correct third term	Allow unsimplified $\frac{3}{4}x^2$ is **A0** unless recovered by later work Expect $\frac{27}{128}x^2$
			$(4+3x)^{\frac{3}{2}} = 8\left(1+\frac{3}{4}x\right)^{\frac{3}{2}} = 8+9x+\frac{27}{16}x^2$	**B1FT**	**1.1a**	Multiply their 3 term expansion by 8	Bracket expanded and coefficients simplified If **B1M1A1** awarded, but attempt to simplify then goes wrong, **B1FT** is **not** also awarded ISW once correct expansion seen
				[4]			
8	**(b)**		$\lvert x\rvert < \frac{4}{3}$ or $-\frac{4}{3} < x < \frac{4}{3}$	**B1**	**1.1**	Could also be $\lvert x\rvert \leq \frac{4}{3}$ or $-\frac{4}{3} \leq x \leq \frac{4}{3}$, as $n > 0$	Must be condition for x, not kx
				[1]			

8	**(c)**		$\left(8+9x+\frac{27}{16}x^2\right)\left(1+2ax+a^2x^2\right)$ coeff of x^2 is $8a^2 + 18a + \frac{27}{16}$	**M1**	**3.1a**	Expand $(1 + ax)^2$ and attempt at least one coeff of x^2	Allow ax as middle term, and/or ax^2 as third term Attempt at x^2 term could be part of a fuller expansion

Question			Answer	Marks	AO	Guidance	
				M1	**1.1**	Attempt all three coeff of x^2, and no others	If part of fuller expansion then **M1** awarded when only three relevant terms used
			$8a^2 + 18a + \frac{27}{16} = \frac{107}{16}$ $8a^2 + 18a - 5 = 0$	**A1**	**3.1a**	Equate to $\frac{107}{16}$ to obtain correct quadratic	aef, including unsimplified **A0** if a mix of terms and coefficients, but can be recovered
			$(2a+5)(4a-1) = 0$ $a = -\frac{5}{2}$ and $a = \frac{1}{4}$	**A1**	**1.1**	Solve quadratic, possibly **BC**, to obtain $a = -\frac{5}{2}$ and $a = \frac{1}{4}$	
				[4]			

Question			Answer	Marks	AO	Guidance	
9	(a)		The rate at which the number of bees changes relative to the rate at which the number of flowers changes or Rate of increase of bees per increase in wildflower plants	**B1** **[1]**	**3.3**	Must mention 'rate of change' of bees, or equiv, and how this relates to wildflower plants	**B0** for 'change' not 'rate of change' **B1** for rate of change of bees with respect to plants **B1** for rate of change of bees as the plants change **B1** BOD for rate of change of bees compared to the **number** of plants State or imply that it is bees compared to flowers, so **B0** if other way around Must relate to bees and plants, and not just B and F See appendix for further examples
9	(b)		$\frac{dF}{dt} = 5e^{0.1t}$	**B1**	**1.1**	Correct derivative No need to see $\frac{dF}{dt}$ notation	Could be unsimplified
			$\frac{dB}{dt} = 2 - 3\sin 3t$	**B1**	**1.1**	Correct derivative No need to see $\frac{dB}{dt}$ notation	NB watch out for $2 - 3\sin t$
			$\frac{dB}{dF} = \frac{dB}{dt} \times \frac{dt}{dF} = \frac{2-3\sin 3t}{5e^{0.1t}}$	**M1**	**3.4**	Correct method to combine their two derivatives – algebraic or numerical	**B0B0M1** is possible Must have $e^{0.1t}$ in denominator, or $e^{-0.1t}$ in the numerator, but allow muddles with the placing of the 5
			$\frac{dB}{dF} = \frac{2-3\sin 12}{5e^{0.4}} = 0.484$	**A1** **[4]**	**3.4**	Substitute $t = 4$ to obtain 0.484, or better	

Question			Answer	Marks	AO	Guidance	
9	(b)		**Alternative method** $t = 10\ln\left(\frac{F}{50}\right)$ $B = 20 + 20\ln\left(\frac{F}{50}\right) + \cos\left(30\ln\left(\frac{F}{50}\right)\right)$	**B1**		Correct expression for B as a function of F	
			$\frac{dB}{dF} = \frac{20}{F} - \frac{30}{F}\sin\left(30\ln\left(\frac{F}{50}\right)\right)$	**M1**		Attempt differentiation	May see ln terms split first (possibly even including use of cos($A - B$)
				A1		Obtain correct derivative aef	
			$\frac{dB}{dF} = \frac{20}{50e^{0.4}} - \frac{30}{50e^{0.4}}\sin\left(30\ln\left(\frac{50e^{0.4}}{50}\right)\right)$ $= 0.484$	**A1**		Substitute $F = 50e^{0.4}$ to obtain 0.484, or better	Could use $t = 4$ if derivative now in terms of t
9	(c)		The data comes from the summer, so taking it beyond 12 weeks is unlikely to be reliable	**B1** **[1]**	**3.5b**	Summer will be over so pattern may not continue	Summer is not greater than 12 weeks Fewer bees and/or flowers in autumn/winter Any reason referring to a change in season having an effect on bees and/or flowers **B0** for just considering long-term behaviour eg flowers will not continue to increase exponentially Reasons must reference seasons / different time of year (could be implied by 'weather getting colder')

Question			Answer	Marks	AO	Guidance	
10	(a)		Both f(0) and f(1) are positive so no sign change will be seen	B1 [1]	2.3	Identify both y-values being positive **and** state 'no sign change' or equiv	Could also evaluate f(0) as 1 and f(1) as 2.9 (or better), and refer to no sign change – these are both positive so no need to include > 0 Could also refer to the asymptote / discontinuity within this range ($x = 0$ to $x = 1$) Also allow 'graph is not continuous in this interval' **B0** for no reference to interval Could say that the two points chosen are not on the same part of the curve
10	(b)		$\frac{e^x}{4x^2-1} = -2$ $e^x = -8x^2 + 2$ $8x^2 = 2 - e^x$	M1	1.1	Attempt rearrangement, as far as $kx^2 = \ldots$	Allow sign error(s) only
			$16x^2 = 4 - 2e^x$ $4x = \sqrt{4 - 2e^x}$ $x = \frac{1}{4}\sqrt{(4 - 2e^x)}$ **A.G.** [2]	A1	1.1	Obtain given answer convincingly	If $x = \sqrt{\frac{1}{4} - \frac{1}{8}e^x}$ then an additional line of working needed before given answer (eg show common denominator of 16)

Question			Answer	Marks	AO	Guidance	
10	(c)		$x_2 = 0.285074813\ldots$	B1	1.1	Correct first iterate (at least 4sf)	State 0.2851 or better
			0.28943, 0.28817, 0.28853, 0.28843, 0.28846, 0.28845...	M1	1.1	Correct iterative process (at least 3 more values)	Allow **M1** for 3sf – expect 0.289, 0.288 and then 0.288 or 0.289 depending whether truncating or rounding
			$\alpha = 0.2885$	A1	1.1	Correct root, given to 4sf, following 2 iterates that agree to 4sf	ie at least 7 iterations needed, given to at least 4sf **A0** for eg $x_8 = 0.2885$ (implies 8th iterate and not root) Process self corrects so **B0M1A1** possible; or **B1M1A1** if error in term other than x_2
				[3]			
10	(d)		$F'(x) = \frac{-16x}{2-8x^2}$	M1	1.1a	Attempt differentiation using the chain rule	Obtain derivative of form $\frac{kx}{2-8x^2}$ Condone subscripts still present in derivative
			$F'(0.3) = -3.75$	M1	1.1	Attempt $F'(0.3)$ – not dependent on previous **M1**, but must follow some attempt at differentiation	**M1** can be implied by correct –3.75 (from correct derivative), but explicit substitution must be seen if $F'(x)$ is incorrect Must come from differentiating $F(x)$ and not a different function

Question			Answer	Marks	AO	Guidance	
			For convergence $\lvert F'(\alpha)\rvert < 1$, but $-3.75 < -1$, so iteration will not find root	**A1** **[3]**	**2.5**	Correct reasoning, following correct $F'(0.3)$	Allow $F'(\alpha) < -1$, hence will not converge Condone $F'(x)$ not $F'(\alpha)$ No credit for just testing the given iterative formula

Question			Answer	Marks	AO	Guidance	
11	**(a)**		$\log_{10}S = \log_{10}(ab^t)$ $\log_{10}S = \log_{10}a + \log_{10}b^t$	**M1**	**3.3**	Attempt to show reduction to linear form	Introduce logs on both sides, and correctly split to the sum of two terms
			$\log_{10}S = t\log_{10}b + \log_{10}a$	**A1**	**3.3**	Obtain correct equation	Condone no base; any bases seen must be 10 **A0** for $\log_{10}bt$ unless previously seen as $t\log_{10}b$
			which is of the form $Y = mX + c$	**A1** **[3]**	**3.3**	Link to equation of straight line	Could instead refer to linear relationship If **M0** then allow **SC B1** for statement such as S against t is an exponential function so log$S$ against t will give a straight line
11	**(a)**		**Alternative method**				
			$\log_{10}S = mt + c$ $S = 10^{mt + c}$	**M1**		Attempt equation of straight line, and attempt expression for S	Must be using $\log_{10}S$ against t Must use base of 10
			$S = 10^{mt} \times 10^c$	**A1**		Correctly split into two terms	
			which is of the form $S = ab^t$	**A1**		Link to exponential model	

Question			Answer	Marks	AO	Guidance	
11	(b)		$m = \log_{10} b = 0.06$ so $b = 10^{0.06} = 1.15$	B1	2.1	Link gradient of line of best fit to linear form and confirm $b \approx 1.15$ Allow m in range [0.055, 0.065]	Or $\log_{10} 1.15 = 0.06$ and compare to gradient
			$c = \log_{10} a = 2.08$ so $a = 10^{2.08} = 120$	B1	2.1	Link intercept of line of best fit to linear form and confirm $a \approx 120$ Allow c in range [2.075, 2.085]	Or $\log_{10} 120 = 2.08$ and compare to intercept
				[2]			Plotted points are linear so may not see line of best fit drawn If substituting into formula (either given model or linear reduction) then **B1** for finding and verifying any point that would be on the line of best fit **B1** for finding and verifying a second point
11	(c)		$S = 120 \times 1.15^7$	M1	3.4	Substitute $t = 7$ into given model	soi
			predicted sales are 319 items	A1	3.4	Conclude with integer value	Accept 320 items
				[2]			

Question	Answer	Marks	AO	Guidance
11 (d) (i)	GP with $a = 138$ and $r = 1.15$	B1	3.1b	State or imply sum of GP with a as 120 or 138, and r as 1.15
	$\frac{138(1-1.15^t)}{1-1.15} = 70000$	M1*	3.1b	Attempt sum of GP, with $a = 120$ or 138 and $r = 1.15$, related to 70000
	$1.15^t = 77.087$	M1 dep*	1.1	Attempt to rearrange equation as far as $1.15^t =$
	$t = 31.088\ldots$ hence 32 months	A1	3.2a	Obtain 32 ('months not required')
		[4]		

Could be implied by attempt to use GP sum formula (but not just n^{th} term) – allow slip as long as clearly sum being considered

Must be correct sum formula
May have n not t throughout
Allow $r = 1.15^{t+1}$ with $a = 120$ but this is **B1 M1** only, as not a valid method)

Must now have $a = 138$ (or equiv)
Allow sign errors only
Allow T&I as not DR

www
If 32 given as **answer only** then allow full marks; if any method shown then mark using the main scheme

Allow BOD with any inequality signs

Question			Answer	Marks	AO	Guidance	
11	(d)	(ii)	Unlikely to be reliable as sales may not continue in same pattern as market could become saturated	B1 [1]	3.2b	State or imply that the model is unlikely to be valid, with a sensible reason why – could refer to reason why pattern may not continue or extrapolation not being reliable	Decrease in demand Increase in competition No values beyond $t = 6$ so pattern unknown Reason why sales are likely to level off / plateau or unlikely to continue to increase ('other factors' not enough) Item sales may vary according to season

Question			Answer	Marks	AO	Guidance	
12	(a)		$du = e^x dx$	B1	1.1	Correct statement linking du and dx	or $dx = \frac{1}{u+2}du$
			$\int \frac{7(u+2)-8}{u^2} \cdot \frac{1}{u+2} du$	M1	1.1	Use $e^x = u + 2$ to attempt integrand in terms of u	Must see clear evidence of substitution, including how $e^x dx$ is dealt with **M0** for going straight from $7e^x - 8$ to $7u + 6$ with no justification Must include du
			$= \int \frac{7u+14-8}{u^2(u+2)} du = \int \frac{7u+6}{u^2(u+2)} du$	A1 [3]	2.1	Correct integrand	Including both integral sign and du throughout, as **AG**

Question			Answer	Marks	AO	Guidance	
12	(b)		$\frac{A}{u}+\frac{B}{u^2}+\frac{C}{u+2}=\frac{7u+6}{u^2(u+2)}$ $Au(u+2)+B(u+2)+Cu^2=7u+6$	M1	3.1a	Attempt correct partial fractions May have $\frac{Au+B}{u^2}+\frac{C}{u+2}$ but **M0** for just $\frac{B}{u^2}$ with no $\frac{A}{u}$	Correct method to combine correct fractions, and at least one constant attempted If considering $\frac{7}{u^2}+\frac{-8}{u^2(u+2)}$ then must use partial fractions on the second term to get credit
			$\frac{2}{u}+\frac{3}{u^2}-\frac{2}{u+2}$	A1	2.1	Correct partial fractions May have $\frac{2u+3}{u^2}-\frac{2}{u+2}$	Possibly implied by their A, B, and C values ie $A = 2$, $B = 3$, $C = -2$
			$2\ln\lvert u\rvert - 2\ln\lvert u+2\rvert - 3u^{-1}$	M1	1.1	Attempt integration of $\frac{B}{u^2}$ and at least one of $\frac{A}{u}$ or $\frac{C}{u+2}$, and no others	Allow errors in coefficients only Allow **M1** if only two fractions, as long as of required form If using $\frac{Au+B}{u^2}$ then it must be a correct integration attempt (ie split into two fractions first)
				A1FT	2.1	FT on their two or three fractions as long as ku^{-2} and one or two fractions each with a linear denominator	Condone brackets not modulus Condone no brackets as long as implied by later working, eg when limits are used
			$(2\ln4 - 2\ln6 - \frac{3}{4}) - (2\ln2 - 2\ln4 - \frac{3}{2})$	M1	1.1a	Attempt use of correct limits – correct order and subtraction; u or x but commensurate with their integral	Allow substitution into any function that is clearly attempt at integration

Question			Answer	Marks	AO	Guidance	
			$\left(\frac{3}{2}-\frac{3}{4}\right)+\ln\left(\frac{4\times4}{6\times2}\right)^2$	**M1**	**3.1a**	Attempt to rearrange correct numerical integral to required form	Must be correct numerical expression from correct working Terms may have been combined before use of limits, but must still be correct expression to gain **M1** Correct attempt to combine ln terms ie deal with coefficients and correct product / quotient for the sum / differences Allow one slip
			$\frac{3}{4}+\ln\frac{16}{9}$	**A1**	**2.1**	Obtain $\frac{3}{4}+\ln\frac{16}{9}$	Condone $\frac{3}{4}+2\ln\frac{4}{3}$ Fractions must be simplified ISW an incorrect attempt to write this answer in a different form, but **A0** if further work done eg multiplying by a constant to clear the fractions
				[7]			

APPENDIX

Exemplar responses for Q9(a)

Response	Mark	Comment
The rate of increase in the number of bees regarding the increase in the number of plants	**B1**	BOD 'regarding'
Rate of bees with respect to flowers	**B0**	no 'change'
Rate of change in number of bees in terms of the number of flowers	**B1**	BOD 'in terms of'
The rate of growth in the number of bees when the number of plants increases over time	**B1**	
The rate of change of flowers according to the number of bees	**B0**	wrong way around
The rate of increase of bees over the rate of increase in wildflowers	**B0**	not 'over' – suggests fraction
The rate of change of number of bees compared to number of plants	**B1**	BOD 'number'
Rate at which the number of bees increases as the number of plants increase	**B1**	Includes 'rate' and 'increases'
The change in number of bees in respect to the number of flowers	**B0**	no 'rate'
The rate of increase in number of bees in accordance to the number of plants	**B1**	BOD 'in accordance'
How the number of bees vary with the number of flowers	**B0**	not rate of change
The rate at which the bees to flowers ratio is changing	**B1**	BOD 'ratio'
Rate of growth of bees depending on the number of flowers	**B1**	
Rate of change of bees per wildflower plant	**B1**	
Rate of change of the number of bees in terms of flowers	**B1**	
The rate of change between the bees and the plants	**B0**	no dependency implied
The rate of change in number of bees against the change in plants	**B1**	
The rate of change of bees compared to flowers	**B1**	
Rate of change of the number of bees as the number of flowers vary	**B1**	
The rate at which the number of bees increase with the number of plants	**B1**	BOD 'with'

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